Problem 8

Let the first Number be 'x' & other be 'y'

Given that $x^2 + y = 48$ — (1)

$\text{Max} = xy = F$

From (1) $x^2 + y = 48 \Rightarrow y = 48 - x^2$

Max $xy \Rightarrow x(48 - x^2) = 48x - x^3$

$F = 48x - x^3$

$F' = 48 - 3x^2$

$F'' = -6x$

A function is Max or Min for $F' = 0$

$\Rightarrow F' = 48 - 3x^2 = 0$

$\Rightarrow x = 4, -4$

If $F'' < 0$; function is Maximum.

$F''(-4) = -6(-4) = 24$

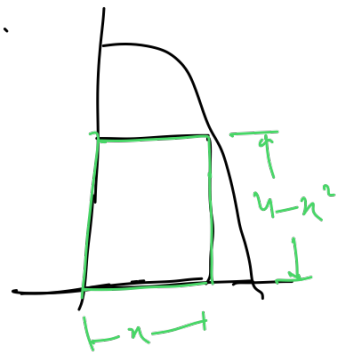
$F''(4) = -6(4) = -24 \Rightarrow$ Maximum for $x = 4$

From eq (1) $y = 48 - x^2$

$= 48 - 16 = 32$

Numbers are $x = 4, y = 32$

Problem 9.



Area of rectangle is given by

$$A = x(4-x^2)$$
$$= 4x - x^3$$

A function is Max or Min $A' = 0$

$$\Rightarrow A' = 4 - 3x^2 = 0 \Rightarrow x = \pm\frac{2}{\sqrt{3}}$$

If $A'' < 0$, A will be Maximum

$$A'' = -6x$$

$$A''\left(-\frac{2}{\sqrt{3}}\right) = -6\left(-\frac{2}{\sqrt{3}}\right) = 4\sqrt{3}$$

$$A''\left(\frac{2}{\sqrt{3}}\right) = -6\left(\frac{2}{\sqrt{3}}\right) = -4\sqrt{3} = \text{Maximum for } x = \frac{2}{\sqrt{3}}$$

Sides of Rectangle are $x = \frac{2}{\sqrt{3}}$; $4 - \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{8}{3}$

$$\text{Max. Area} = \frac{2}{\sqrt{3}} \times \frac{8}{3} = \boxed{\frac{16\sqrt{3}}{9}}$$

Problem 10

$$R = cW(21 - W)$$

$$R = 21cW - cW^2$$

A function is Max or Min $R' = 0$

$$\Rightarrow R' = 21c - 2cW = 0 \Rightarrow W = \frac{21}{2}$$

If $R'' < 0$; R will be Maximum

$$R'' = -2c < 0$$

Hence $W = \frac{21}{2} = 10.5$ Pounds ; growth rate is largest.